Vertical integration smooths innovation diffusion*

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Abstract

Does vertical integration of an input innovator with a downstream firm entail innovation foreclosure? We study the licensing incentives of an independent input producer owning a patented product innovation which allows the downstream firms to improve the quality of their final goods. We consider two-part tariff contracts for both outside and incumbent innovators. We find that the incumbent innovator has always the incentive to license its innovation to the rival firm so that under vertical integration complete technology diffusion takes place. In contrast, the external patent holder may prefer exclusive licensing depending on the innovation size as well as on the set of allowed contracts. As a result vertical integration does not entail innovation foreclosure, rather it facilitates innovation diffusion with respect to vertical separation.

As for the profitability, the vertical integration with either downstream firm is always privately profitable and it is welfare improving for large innovations: this implies that not all profitable mergers should be rejected.

Keywords: Patent licensing, product innovation, two-part tariff, negative royalties, vertical differentiation, vertical integration.

JEL Classification: L15, L13, L24.

1 Introduction

We develop a theoretical model to study the incentives of an independent input innovator to diffuse its innovation either through licensing or through vertical integration. We consider input innovations that improve the quality of the final good and/or possibly create vertical differentiation in the product market.

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More precisely, we consider a standard vertical product differentiation model (Gabszewicz and Thisse, 1979, Mussa and Rosen, 1982) and we analyse two downstream firms producing and selling a final output to heterogeneous consumers, and two differentiated inputs in the upstream market, a low quality input provided by competitive firms and a high quality patented input provided by an independent input innovator. The quality of the final good depends on the quality of the input. Complete technology diffusion implies a homogeneous final good of high quality, whereas exclusive licensing implies a vertically differentiated market. We consider a general two-part tariff contract for both outside and incumbent innovators. In particular, the set of possible contracts, that are observable, is such that the fixed fee has to be non-negative, whereas the per-unit royalty might also be negative, that is, it could be a per-unit subsidy. The motivation for such assumption comes from the observation that, while negative fixed fees would be clearly held to be illegal by antitrust authorities as they could be a means to strand the rival firm out of the market, a per-unit subsidy cannot *a priori* be considered welfare detrimental. We endogenize market structure allowing the patent holder to vertically integrate with either downstream firm. Formally, we develop the following game. First, the innovator decides whether to enter the market through vertical integration. Second, in case of external patent holder, the innovator offers a contract to each downstream firm on a take-it-or-leave-it basis; whereas, in case of internal patent holder, the vertically integrated innovator offers a contract to the rival. Third, the potential licensees decide whether to accept or reject the contract. Finally the downstream firms compete.

Main results. We find that the incumbent innovator, that is, when the patent holder decides to enter the market producing with the new input, has always the incentive to license its innovation to the rival firm; so that under vertical integration complete technology diffusion takes place. In contrast, the external patent holder, that is, when the innovator decides to stay vertically separated, may prefer exclusive licensing depending on the innovation size as well as on the set of allowed contracts. Namely, in case of negative royalties, exclusive licensing always prevails; in contrast, when negative royalties are not allowed, complete technology diffusion occurs for small innovations, whereas exclusive licensing does for large innovations. As a result, vertical integration facilitates innovation diffusion. At first glance these incentives might seem counterintuitive. Indeed, one could expect the external patent holder to prefer complete technology diffusion because licensing is its only source of profit and the internal patent holder to prefer innovation foreclosure in order to benefit from its competitive advantage.

The insights are as follows. An incumbent patent holder always licenses the product innovation thus preferring a homogeneous high quality good market rather than a vertically differentiated market in which the internal patentee would be the high quality firm competing with a low quality producer. By setting a positive royalty level and so raising its rival cost, the incumbent innovator gains most of the market and leave the rival with its outside option. This way it is able to reach a quasi-monopoly outcome.

As for the outside inventor, in case of *negative royalties*, he optimally sells an exclusive license via a two-part tariff contract that specifies a per-unit subsidy: the innovator via the subsidy makes the unique licensee more aggressive in the Cournot market and reap higher profit via the fixed fee (Fershtman and Judd, 1987). An alternative explanation is provided by Jing and Winter (2013) that referring to Marvel (1982), point out that "an exclusivity contract can protect investments by an upstream supplier against free-riding by competitors on investments such as training that the supplier makes in retail relationships."

Instead, in case of *non-negative royalties*, the technology diffusion depends on the innovation size. In particular, the optimal contract under exclusive licensing only specifies a positive fixed fee, whereas under complete technology diffusion the optimal contract specifies both a positive royalty and a positive fixed fee. Under both exclusive licensing and complete technology diffusion, the external patent holder's profits increase with the innovation size. However, the profit from exclusive licensing increases more than the profit from complete technology diffusion so that when the innovation is high enough exclusive licensing becomes more profitable than complete technology diffusion. The reason for this result is twofold. First, under exclusive licensing the patent holder has to care about only one outside option, whereas under complete technology diffusion it has to care about two outside options. The higher is the innovation size, the lower is the firms' outside option and, in turn, the larger is the profit share that the patent holder can get; in other words, the fixed fees are increasing in the innovation size. Under exclusive licensing this is the unique effect at work, given that the optimal per-unit royalty is zero. In contrast, under complete technology diffusion, there is a second mechanism. In this case the profit has two components: the fixed fees and the revenues coming from the royalties. Disentangling this latter component, it is easy to verify that the equilibrium per-unit royalty increases but the equilibrium quantities produced by firms decrease with the innovation size. Whereas the overall effect is always positive, the presence of this negative effect combined with the presence of two outside options under complete technology diffusion explains why the profit under exclusive licensing is larger than the profit under complete technology diffusion for sufficiently high values of the innovation size.

As far as the *merger profitability* is concerned, under Cournot competition the vertical integration of the upstream inventor with either downstream firm is always privately profitable. This result is in line with the new market foreclosure theory (see Rey and Tirole 2007) according to which vertical integration allows the monopolist upstream producer to protect its monopoly power. As for the *social profitability*, we find that the merger is also welfare improving for large innovations; this implies that not all profitable mergers should be rejected. Indeed, on one hand, the merger ensures the innovation diffusion and pushes prices down as it implies the (partial) internalization of the vertical externality; on the other hand, the merger has an anticompetitive effect because the vertically integrated firm is able to (partially) foreclose the rival firm via a positive per-unit royalty. The first two positive effects prevail as long as the quality improvement associated with the innovation is sufficiently large. Thus, a very simple prescription arises: the antitrust authority should approve mergers where large product innovations are involved.

Empirical evidence. There are several examples of innovation in input required for the production of final goods that result in an improved quality. The introduction of a faster computer chip; the wheat bread flour made in the stone mill as 200 years ago;[1] the Loro Piana innovation in new materials (super-luxury natural textiles).[2]

As for the *diffusion incentives*, we can find empirical support to our results in the agrochemical and in the pharmaceutical sectors, for instance. In these markets, it is quite usual to observe mergers and acquisitions of innovative companies (external research laboratories) from big firms that then license the patented product to the other firms. In 1996, Monsanto purchased Agracetus, the biotechnology company that had generated the first transgenic varieties of cotton, soybeans, peanuts, and other crops, and from which Monsanto had already been licensing technology since 1991.[3] The patent on the first type of Roundup Ready crop that Monsanto produced (soybeans) has been broadly licensed to other seed companies (about 150 companies, including Syngenta and DuPont Pioneer).[4] Most of Monsanto's annual sales come from seeds, increasingly of genetically modified (GM), or transgenic, varieties, and from licensing genetic traits.[5] Similarly "the Muñoz Group had worked steadily to integrate vertically -...- in an attempt to control its produce business from seed to plant to retailer..." In many cases this firm licenses its patented products to grower-partners.[6] Nespresso provides another example of internal patent holder selling its own patented coffee machines and capsules in the market but also licensing them to other retailers. The external patent holder may instead decide not to completely diffuse the new product, especially when dealing with large innovations. Some luxury or semi-luxury goods that are sold only by one retailer in a town fit our theoretical result. Methius wine is produced in Trentino Alto-Adige and, in Milan, for instance, it is purchasable only in the wine shop Cotti.

The soybean seed market makes a case study of *vertical integration incentives*. As pointed out by Shi and Chavas (2011), recent advances in biotechnology have led agricultural biotech firms (who produce the seeds sold to farmers) to differentiate their seed products through patented genetic materials. The vertical organization of this industry has changed. While biotech firms producing patented genes have relied extensively on licensing their technologies to seed companies, they have recently increased their use of vertical coordination through integration. Shi and Chavas (2011) use a Cournot model of multi-

[1] It is a case of reswitching of techniques to traditional Italian whole wheat breads made 200 years ago when the local stone milled flour was the only option.

[2] This thread starts with Tasmanian wool in the 1970s, through vicuna (LP was granted 10-year exclusivity), baby cashmere to, most recently, lotus flower cloth (i.e., the fibers of nelumbo nucifera, an aquatic perennial more commonly known as the lotus).

[3] See BOCA RATON, Fla., April 8,1996 – W. R. Grace & Co. (NYSE: GRA).

[4] See Monsanto.com November 3, 2008 and Monsanto GMO Ignites Big Seed War. NPR.

[5] See http://www.economist.com/node/14904184.

[6] Harvard Business School case, the Muñoz Group.

product firms and find that seeds sold through vertically-integrated structures are priced higher than those that are licensed.

Nevertheless, the merger guidelines do not exclude that vertical merger could increase social welfare for efficiency reasons. For instance, the European Commission mentions in its recent Guidelines on the assessment of non-horizontal mergers (2008): "The Commission may decide that, as a consequence of the efficiencies that the merger brings about, there are no grounds for declaring the merger incompatible with the common market pursuant to Article 2(3) of the Merger Regulation. This will be the case when the Commission is in a position to conclude on the basis of sufficient evidence that the efficiencies generated by the merger are likely to enhance the ability and incentive of the merged entity to act pro-competitively for the benefit of consumers, thereby counteracting the adverse effects on competition which the merger might otherwise have". In contrast, the Swiss Competition Commission forces the (vertically integrated) Swatch Group to sell its watch components to the competitors.[7]

As for the *licensing contract*, from an empirical point of view, per-unit subsidies are rarely observed, however they might be present implicitly in the form of transmission of know-how and technical assistance to the licensee (Liao and Sen, 2005). There are examples in the personal computer industry that might fit this scenario. Gawer and Henderson (2007) explore Intel's strategy with respect to complements. They find that Intel, as provider of microprocessors, an essential input of the personal computer, may have the incentive to subsidize complements' production by the development and widespread dissemination of intellectual property.

Related literature. This paper contributes to the literature on licensing a product innovation as well as to the debate on the competitive effects of vertical integration.

Regarding the literature on optimal licensing, most papers focus on cost-reducing, *process innovations* (see Kamien and Tauman, 1986; Katz and Shapiro, 1986; Kamien, Oren and Tauman, 1992; Sen and Tauman, 2007; Erutku and Richelle, 2007). Other papers investigate the issue of licensing a *product innovation*.[8] A first contribution is by Kamien, Tauman and Zang (1988). They consider the introduction of a new product in a Cournot oligopoly focusing on the fixed fee licensing mode. More recently, Lemarié (2005) compare fixed fee versus royalty for a demand-enhancing innovation. Our departure from this literature is threefold. First, we consider an innovation that improves the quality of a product.[9] Second, we extend the analysis to the case of negative per-unit royalties, whereas the focus is generally on contracts that do not allow for per-unit subsidies. Finally, we endogenize the role of the innovator by letting it

[7] Harvard Business School case, "The Swatch Group".

[8] As pointed out by Kamien et al. (1988) "a product innovation can be regarded as a cost reducing innovation by assuming that the new product could have been produced before but with a sufficiently high marginal cost that rendered its production unprofitable."

[9] Stamatopoulos and Tauman (2008) analyse optimal licensing for an outside inventor of an innovation that improves the quality of a product and also affects its marginal cost (the innovation can be classified as product and process at the same time). They consider the logit demand framework with price competition.

decide whether to enter the market, whereas typically the innovator is exogenously assumed to be either an incumbent or an external patent holder.

As far as the *non-negative royalty case* is concerned, in line with the theoretical result by Sen and Tauman (2007), we find that the *external patent holder* optimally sells an exclusive license via a fixed fee, when the innovation is very large, and sells two licenses via a positive per-unit royalty as long as the innovation is not too small, thus fitting the wide prevalence of per-unit royalties over fixed fee in practice (see for instance, Rostoker (1984)).

As for the *internal patent holder*, complete technology diffusion takes place and the optimal contract is a pure royalty policy. Again, this result is line with Sen and Tauman (2007).[10] Arya and Mittendorf (2006) study the incentives of an internal patent holder to license a product innovation in the presence of a monopolist input producer. They find that the licensing incentive is positive as it affects the input pricing terms in a way that is beneficial for the internal patent holder even though it implies giving up the monopoly power on the final good. We also find that the internal innovator has incentive to license the new good to the rival, however, the reason is to extract some surplus from the rival firm that is active in the market independently of the licensing strategy.

Concerning the *negative royalty case*, to the best of our knowledge the main contributions are Liao and Sen (2005) and Milliou and Petrakis (2007). Our paper is more closely related to Liao and Sen (2005) that study optimal licensing of a process innovation in a Cournot duopoly with homogeneous goods. They show that subsidy-based contracts are optimal under exclusive licensing but not under complete technology diffusion. They conclude that negative royalties are welfare improving with respect to non-negative royalties if the innovation is sufficiently small. We depart from this paper in what we endogenize market structure and we find that vertical integration is profitable independently of whether negative royalties are allowed.

Finally, as for the *competitive effects of vertical integration*, the new market foreclosure theory points out that vertical integration can restrict downstream competition. In particular in the recent survey by Rey and Tirole (2007) it is shown that vertical integration is always profitable as the upstream producer can restore its monopoly power through input foreclosure, but vertical integration is anticompetitive. Reisinger and Tarantino (2013) extend the theory of Rey and Tirole (2007) by analysing the profitability of vertical integration in the presence of complementary input producers and show that vertical integration can be privately unprofitable (for earlier works on complementary inputs see Arya and Mittendorf (2007) and Laussel (2008)). In particular, vertical integration raises the market profit of the merging entity and it is therefore profitable; however, the presence of a complementary input implies that part of this larger profit can be extracted by the supplier of this input. This expropriation effect can render vertical integration unprofitable. In our framework vertical integration remains privately profitable but the social welfare profitability depends on

[10] There are also previous contributions about an internal patent holder, see *inter alia* Wang (1998 and 2002).

the innovation size. Our results contrast with Sandonís and Faulí-Oller (2006) that consider non-drastic process innovations in a horizontally differentiated Cournot duopoly, and find that the merger is privately profitable only for small innovations and all profitable mergers are welfare detrimental. They point out the commitment problem faced by the vertical merger (that is the insider innovator) which has only one instrument, the licensing contract to the rival firm rather than two (a licensing contract to each downstream firm), and cannot credibly restrict its output as the new input is transferred at marginal cost. In our model, this result breaks down as the incentive to diffuse the innovation makes homogeneous the downstream market. Another related paper on the implication of vertical integration is Milliou and Petrakis (2012) that consider an upstream monopolist selling an essential input to two competing asymmetric downstream firms. They study the upstream monopolist's incentive to vertically integrate with the more efficient downstream firm given that vertical integration creates the possibility of knowledge disclosure, and they analyse the integrated firm's incentive to disclose it to its downstream rival. They find that under vertical integration knowledge is disclosed, firms' innovation incentives, consumer and total welfare increase, whereas the rival's cost decreases. Even though in line with our results on the diffusion incentives of vertical integration, Milliou and Petrakis (2012) abstract from licensing and they focus on linear pricing where the two downstream firms differ in their constant marginal cost of production so that under vertical integration knowledge disclosure to the rival entails the diffusion of a sort of process innovation.

Recently, Chambolle et al. (2015), Milliou and Pavlou (2013) and Schmidt (2014) even though considering different models, point out somewhat similar innovation gains coming from mergers in vertically related industries. Chambolle et al. (2015) consider a monopolist retailer's incentive to develop a premium private label either through outsourcing or through vertical integration. They find that vertical integration induces more quality investment than outsourcing because of a hold-up effect.[11] The other two contributions instead focus on the efficiency gains of horizontal mergers. Milliou and Pavlou (2013) analyse a bilateral duopoly and study the effects of upstream mergers on R&D investments. They find that mergers can induce an increase in R&D investments and in turn efficiency gains that are passed on to consumers. Finally, Schmidt (2014) considers complementary patents owned by different IP holders and necessary to produce a final good. In this framework two external effects arise: the complements effect that could be solved by horizontal merger and the double mark-up effect that could be solved by vertical merger. He compares the two business strategies of vertical and horizontal integration concluding in favor of the latter as a way to stimulate innovation.

The remainder of the paper is structured as follows. In Section 2, we first set

[11] For details on the spread of private labels products and buyer power of large retailers see Inderst (2013). He analyses a bilateral monopoly and studies the innovation activity of both the manufacturer and the retailer. He isolates two effects: a standard hold-up problem for the manufacturer and a rent appropriation motive for the retailer. The result is that the innovation activity may inefficiently switch to a large retailer when it attains sufficient size.

up the ante-innovation model and we then introduce the product innovation. In Sections 3 and 4 we study the optimal licensing of an external and an internal innovator, respectively. In Section 5, we endogenize the vertical integration decision by comparing the private incentives of the patent holder; we finally compare the social incentives of vertical integration versus vertical separation. In Section 6, we discuss the main assumptions of the model. Section 7 concludes. The formal details (the solutions of the subgames) and the proofs of our results are relegated to the Appendix.

2 Model

2.1 Status quo

We consider two firms producing a homogeneous good and competing à la Cournot. Final output production requires an essential input provided by a competitive upstream market.

As far as the *demand side* is concerned, we assume that there is a continuum of consumers indexed by θ which is uniformly distributed in the interval $[0,1]$. Thus, θ is a taste parameter. Each consumer has a unit demand and buys either one unit of a good of quality s at price p or buys nothing at all. Consumer's utility takes the following form:

$$U(\theta) = \begin{cases} \theta s - p, \text{ if consumer type } \theta \text{ buys} \\ 0, \text{ if does not buy} \end{cases}$$

The demand for the good is then

$$Q(p) = 1 - \left(\frac{p}{s}\right) \iff p(Q) = s(1-Q), \tag{1}$$

where $Q = q_1 + q_2$ and p/s is the fraction of consumers with a taste parameter less than θ, that is the fraction of consumers not buying the good.[12] For future reference we define the consumer surplus as

$$CS(s) = \int_{\frac{p}{s}}^{1} (\theta s - p)\, d\theta = \frac{(p-s)^2}{2s} \tag{2}$$

As for the *supply side*, the essential input of quality s is produced at zero fixed cost $f_L = 0$ and at constant marginal cost $c = 0$ and it is sold at the competitive price $w = 0$. In this framework quality is assimilated to input. The downstream (D) firm i profit function is: $\pi_i = pq_i$. D firms compete in the

[12] At equilibrium the market is uncovered, that is $Q < 1$.

quantities, then the Cournot duopoly equilibrium is:

$$q_i\left(c_i=0,c_j=0;s,s\right)=\frac{1}{3},Q=q_1+q_2=\frac{2}{3} \tag{3}$$

$$p=\frac{s}{3}$$

$$\pi_i=\frac{s}{9},PS=\frac{2}{9}s \tag{4}$$

$$CS=\frac{2}{9}s,SW=CS+PS=\frac{4}{9}s$$

The D firms' price and profits depend on the quality of the input s. Q denotes total output, which corresponds to the demand for the input faced by the upstream (U) market (perfect vertical complementarity).[13]

2.2 Innovation

Suppose that an independent input producer obtains a patented product innovation which allows the downstream firms to improve the quality of their final goods. In the upstream market there is now a monopolist selling an input that ameliorates final product quality by $\psi>1$ that measures the innovation size; so that with this innovation the quality goes from s to $s\psi$. Assume that the invention of the new high quality input entails a fixed cost $f_H=f>0$. Apart from this, production costs are then zero.[14]

We study the licensing incentives of this patent holder. The U firm can sell the new input either to one or both D firms via a two-part licensing contract (r,F). We consider the set of possible contracts such that $r\in\mathbb{R}$ and $F\geq 0$. Indeed, whereas negative fixed fees would be clearly held to be illegal by antitrust authorities,[15] a negative per-unit royalty, that is, a per-unit subsidy (given that the marginal cost is equal to zero), cannot *a priori* be considered welfare detrimental. Some recent contributions on licensing point out the private incentives to subsidize the downstream production (Liao and Sen (2005) and Milliou and Petrakis (2007)), that in some cases are also welfare improving.

In order to determine the upfront fee of the two-part tariff contract we assume that once the innovator chooses the number of licenses $m=\{1,2\}$ to offer and the per-unit royalty r, the fee is determined through a first-price sealed-bid auction and m highest bidders win the license.[16] This is the so called

[13] Note that we could also consider a market interval $[a_1,a_2]$ with $a_2>a_1>0$, however this would require conditions on the parameters to be satisfied at equilibrium to ensure that the market is either covered or uncovered. In particular, in case of covered market, Cournot competition would be constrained; whereas in case of uncovered market the equilibrium results would be qualitatively the same.

[14] We discuss about this assumption in Section 6.

[15] As you will see from the following equilibrium analysis, in fact, it emerges that our restriction to non negative fees is binding only for the internal patent holder, that, if allowed, would induce the nonintegrated firm to produce a nil quantity (foreclosure) by compensating it for the outside option.

[16] In case of tie, that is relevant only under exclusive licensing, we assume, without loss of generality, that firm 2 innovates.

auction plus royalty (AR) policy.[17]

According to the U innovator licensing incentives, different cases derive:

1. *Complete technology diffusion*: all D firms adopt the new input and we have a homogeneous final good of quality $s\psi > s$. D firms' profits $\pi_i\left(c_i, c_j; s\psi, s\psi\right)$ depends on the two part-tariff contracts $c_i = (r_i, F_i)$ with $i = 1, 2$ and $i \neq j$.

2. *Exclusive licensing*: only one of the D firms adopts the new input and we have two final goods of different qualities. The non-innovating firm, say firm 1, produces the low quality good thus incurring zero production costs and gains $\pi_1\left(0, c_2; s, s\psi\right)$; while the innovating firm 2 produces the high quality good and gets $\pi_2\left(c_2, 0; s\psi, s\right)$.

Notice that this formulation of the model entails that input production is carried out solely by the patent holder and that obtaining a license implies for the firm the right to receive the high quality input from the patent holder. This is slightly different from a standard patent licensing model where a licensee produces the good itself after receiving the innovation. However, our assumption of zero variable production costs makes equivalent these two perspectives (whether, once sustained the fixed cost to invent the new input, this is produced by the patent holder or by the licensee(s)).

3 Vertical separation (external patent holder)

Under vertical separation the innovator stays out of the market. We develop the following three-stage game: first, the innovator offers a contract to each D firm on a take-it-or-leave-it basis; second, the potential licensees decide whether to accept or reject the contract; finally the D firms compete. Note that all the contracts are observable.[18] Two possible subgames derive from the innovator's licensing incentives: either exclusive licensing or complete technology diffusion. Under exclusive licensing, suppose only firm 2 adopts the new input so that firm 2 sells the high quality good at price p_2 and firm 1 sells the low quality good at price p_1. Consumer type θ utility takes the following form:

$$U(\theta) = \begin{cases} \theta s\psi - p_2, \text{ if consumer type } \theta \text{ buys the high quality good,} \\ \theta s - p_1, \text{ if consumer type } \theta \text{ buys the low quality good,} \\ 0, \text{ if does not buy.} \end{cases}$$

From the above formulation of the utility function, we can define the consumer indifferent between buying the low quality good and not buying at all, and the

[17] Alternatively, one could consider the upfront fee plus royalty (FR) policy, that is such that the innovator chooses the number of firms to whom the license is offered, the per-unit royalty and the fixed fee that each licensee has to pay (in this case the innovator makes a binding commitment that it will not sell the innovation to any firm outside the chosen subset). However, it can be proved that the patent holder always prefers the AR policy over the FR policy. See Liao and Sen (2005).

[18] This is an usual assumption in patent licensing.

one indifferent between buying the low quality good and the high quality good, as follows:

$$\begin{aligned}\underline{\theta} &= \frac{p_1}{s}, \\ \widehat{\theta} &= \frac{p_2 - p_1}{s(\psi - 1)}.\end{aligned}$$

We focus on the case in which both firms are active in the market, i.e., $0 < \underline{\theta} < \widehat{\theta} < 1$, that is, $p_2 \in (p_1\psi, p_1 + s(\psi - 1))$, which is non-empty for $p_1 < s$. More precisely, in order to find the Cournot equilibrium, we proceed as follows. We assume that the demands for both goods are positive (formally, this means that $0 < \underline{\theta} = \frac{p_1}{s} < \widehat{\theta} = \frac{p_2 - p_1}{s(\psi-1)} < 1$), in the Appendix (8.1), where we solve for the Cournot equilibrium, we derive the corresponding candidate equilibrium and we finally check whether this is the effective equilibrium. The demands for the goods, in this price region, are then:[19]

$$q_1 = \widehat{\theta} - \underline{\theta} \tag{5}$$

$$q_2 = 1 - \widehat{\theta}. \tag{6}$$

Whereas under complete technology diffusion both firms sell the high quality good at price p^{ψ}. Consumer type θ utility takes the following form:

$$U(\theta) = \begin{cases} \theta s\psi - p^{\psi}, \text{ if consumer type } \theta \text{ buys,} \\ 0, \text{ if does not buy.} \end{cases}$$

So that the demand for the good, in the price region $p^{\psi} < s\psi$, is defined as:[20]

$$Q\left(p^{\psi}\right) = 1 - \left(\frac{p^{\psi}}{s\psi}\right).$$

Solving backwards, we find the Nash equilibrium for the external patent holder subgame, that is, its optimal licensing incentives, that we gather in the following Proposition.

Proposition 1 *If* negative royalties *are allowed, the external patent holder prefers exclusive licensing. The optimal contract is:*

$$r_2^{ELneg} = -\frac{s}{2} < 0, F_2^{ELneg} = \frac{\left(4\psi^2 + 1\right)s}{4(4\psi - 1)}.$$

If instead negative royalties are not allowed, the external patent holder prefers exclusive licensing for a sufficiently high product innovation, namely $\psi > \overline{\psi} =$

[19] Outside this price region, either both firms do not produce or only one firm produces a positive quantity: for these pairs of prices, demand functions (5) and (6) do not apply. In the Supplementary material, we provide the demand functions for every pair of non-negative prices.

[20] In the Appendix, we verify that, at equilibrium, $\frac{p^{\psi}}{s\psi} < 1$.

1.8, *and complete technology diffusion otherwise. The optimal contract under* non-negative royalties *is: for* $\psi > \overline{\psi}$,

$$r_2^{EL} = 0, F_2^{EL} = \frac{(\psi - 1)\, s\psi}{(4\psi - 1)},$$

for $\psi \leq \overline{\psi}$,

$$\begin{cases} r_1 = r_2 = r^T = \frac{\left(s\psi - 26s\psi^2 + 16s\psi^3\right)}{64\psi^2 - 14\psi + 4},\ F^T\left(r^T, r^T\right) = \frac{\left(248\psi^2 - 17\psi - 272\psi^3 + 256\psi^4 + 1\right)s\psi}{4(32\psi^2 - 7\psi + 2)^2} \text{ if } \psi \in \left[\psi^T, \overline{\psi}\right], \\ r_1 = r_2 = 0,\ F^T(0,0) = \frac{(\psi-1)(16\psi-1)s\psi}{9(4\psi-1)^2} \text{ if } \psi < \psi^T = 1.585\,6. \end{cases} \quad (7)$$

Proof. See Appendix (8.3). ∎

Under the case of *negative royalties*, the innovator optimally sells an exclusive license via a two-part tariff contract that specifies a per-unit subsidy: the innovator via the subsidy makes the unique licensee more aggressive in the Cournot market, that is subsidization allows the innovator to conquer most of the downstream market by selling the high quality good and reap higher profit via the fixed fee (Fershtman and Judd, 1987). Liao and Sen (2005) consider non-drastic process innovations in a Cournot duopoly with homogeneous goods and find similar results for the diffusion incentives of the external patent holder, except that in our product innovation framework, exclusive licensing prevails also under non-negative royalties, for the case of a large enough product innovation.

As for the case of *non-negative royalties*, the technology diffusion as well as the optimal contract (in particular the positiveness of the royalty) depends on the innovation size. When the innovation is very small, the inventor's incentive is to sell two licenses via a per-unit price as low as possible, that is the optimal contract is a fixed fee. In other words, given the positive outside option of producing the low quality good at zero marginal cost, for very small innovations the inventor cannot set a positive per unit royalty. For intermediate innovations we have again complete technology diffusion but with a positive per-unit royalty. Finally, for large innovations, exclusive licensing takes place. This result could resemble what happens under optimal licensing of process innovations, where the cost-reducing innovation is sold to only one firm via a fixed fee when it is drastic. Note however, that it is not possible in our scenario to distinguish between drastic and non-drastic product innovations, as for any innovation size ψ both firms attain to stay in the market at equilibrium. In order to get an intuition for this result, it is useful to go through the different components of the patent holder profits. First, notice that in both cases of technology diffusion, licensee i's outside option is the same and it is equal to $\pi_i(0, r_j; s, s\psi)$.[21] Under exclusive licensing the patent holder has to care about only one outside option

[21] As detailed in the Appendix, under exclusive licensing, assuming that the patent holder offers the innovation to firm 2 (and to firm 1 if firm 2 does not accept), its outside option is $\pi_2(0, r_1; s, s\psi) = \frac{(s\psi + r_1)^2}{(4\psi - 1)^2 s}$, and under complete technology diffusion it is $\pi_1(0, r_2; s, s\psi) = \frac{(s\psi + r_2)^2}{(4\psi - 1)^2 s}$, given that we reasonably assume $r_1 = r_2$.

whereas under complete technology diffusion it has to care about two outside options. This outside option is decreasing in ψ: the higher is the innovation size, the lower is the firms' outside option and in turn the larger is the profit share that the patent holder can get; in other words, the fixed fees are increasing in ψ. Under exclusive licensing this is the unique effect at work given that the optimal per-unit royalty is zero. In contrast, under complete technology diffusion, the profit has two components: the fixed fees and the revenues coming from the royalties. Disentangling this latter component, it is easy to verify that as ψ increases, the equilibrium per-unit royalty increases but the equilibrium quantities produced by firms decrease.[22] Whereas the overall effect is always positive, the presence of this negative effect combined with the presence of two outside options under complete technology diffusion explains why the profit under exclusive licensing is larger than the profit under complete technology diffusion for sufficiently high values of ψ.[23]

Furthermore, the following remark is worthy.

Remark 2 *As the two-part tariff contracts offered to each D firm are observable, the U monopolist could implement the monopoly outcome. However, the U monopolist has not incentive to implement this outcome.*

The U monopolist could indeed set a royalty such that each firm produces half monopoly quantity:

$$\begin{aligned} q_1 &= \frac{1}{3s\psi}\left(s\psi - 2r_1 + r_2\right) = \frac{1}{4}, \\ q_2 &= \frac{1}{3s\psi}\left(s\psi - 2r_2 + r_1\right) = \frac{1}{4} \end{aligned}$$

$\Longleftrightarrow r_1 = \frac{1}{4}s\psi, r_2 = \frac{1}{4}s\psi$. This implies the following total quantity, price, and industry profit: $Q^m = \frac{1}{2}$, $p^m = \frac{1}{2}\psi s$, $PS^m = \frac{1}{4}\psi s$. So that U profit is given by the per-unit royalty times the quantity produced, plus the fees received by the licensees: $\Pi_U^m = \frac{1}{4}s\psi\frac{1}{2} + 2\left(\frac{1}{16}\psi s - \frac{25}{16}\left(4\psi - 1\right)^{-2}\psi^2 s\right) = s\psi\frac{-41\psi+32\psi^2+2}{8(4\psi-1)^2}$.[24] However the U monopolist has not incentive to implement this outcome because of the presence of the D firms' outside options. More precisely, comparing this profit with the equilibrium profits obtained with the contracts identified in Proposition 1, (expressions 16 and 17 of profits with negative and positive royalties provided in the Appendix) we find that this contract to implement the monopoly outcome is not profitable, even if side payments (F lower than 0) were allowed.[25]

[22] $\frac{\partial}{\partial\psi}r^T > 0$ and $\frac{\partial}{\partial\psi}q^T\left(s\psi\right) < 0$.

[23] Formally, for $\psi > \psi^T$, $\frac{\partial}{\partial\psi}\Pi_U^{EL} > \frac{\partial}{\partial\psi}\Pi_U^T > 0$. The equilibrium expressions for the profits, producer surpluses (industry profits) and consumer surpluses are provided in the Appendix (8.2).

[24] Notice that this profit is positive iff $\psi > 1.23$.

[25] There is also another contract that could implement the monopoly outcome:

$$r = 0, F = \frac{\pi_H^m(0)}{2} - \pi_i\left(0, 0; s, s\psi\right) = \frac{\psi s}{8} - \frac{\psi^2 s}{\left(4\psi - 1\right)^2} = \frac{\left(16\psi^2 - 16\psi + 1\right)s\psi}{8\left(4\psi - 1\right)^2}$$

Finally comparing the agents' payoffs and social welfares before and after the introduction of the innovation, we conclude the following results.

Corollary 3 *As a result of the innovation: consumers are better off and the licensees (the downstream firms) are worse off with respect to the status quo; as long as the fixed cost is sufficiently low, the social welfare increases.*

Proof. See Appendix (8.4). ■

It is worth emphasizing that the licensees are worse off with respect to the status quo, in both cases of exclusive licensing and complete technology diffusion. Interestingly, under complete technology diffusion they face a prisoner's dilemma: accepting the innovation is a weakly dominant strategy for both firms, however they would be better off by not accepting the innovation.[26]

Corollary 4 *Whenever the patent holder is outside the market, allowing for negative royalties is welfare improving for any innovation size.*

Proof. See Appendix (8.5). ■

Although in case of negative royalties the innovation is not completely diffused, thanks to production subsidizing consumer surplus and total welfare turn out to be higher under exclusive licensing. The increase of output induced by the per-unit subsidy overcompensates the lower average quality in the market.[27] Liao and Sen (2005) show that negative royalties are welfare improving with respect to non-negative royalties if the innovation is sufficiently small. They point out the importance of the cost of exclusion: the additional cost that the non-licensee firm has to pay under exclusive licensing. As long as this cost is low, exclusive licensing (that arises with negative royalties) is preferred with respect to complete technology diffusion.

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where $\pi_H^m(0)$ is the monopoly profit in case of high quality good and zero marginal cost and the outside option is not buying the innovation given that the rival does. In this case U gets $\Pi_U = 2\left(\frac{(16\psi^2-16\psi+1)s\psi}{8(4\psi-1)^2}\right)$, the D firms get: $\pi_1 = \pi_2 = (4\psi-1)^{-2}\psi^2 s$ (which is incentive compatible). Comparing this contract with (7) from U viewpoint we obtain that this contract is superior. However this is not credible because when U offers a zero per-unit royalty, each D firm has the incentive to deviate from half monopoly quantity (prisoner's dilemma) knowing that the rival has zero marginal cost and produces half monopoly quantity. Indeed by deviating it would be able to pay the fixed fee and to get higher profit. The result of the one-shot game would be that they produce the duopoly quantity and they are not able to pay the fixed fee.

[26] Formally, this is evident by considering the second stage of the licensing game in normal form, where the row player is firm 1 and the column player is firm 2, their strategy is either A (accept) or N (not):

	A	N
A	$\pi\left(0, r^T; s, s\psi\right), \pi\left(0, r^T; s, s\psi\right)$	$\pi_1\left(r^T, 0; s\psi, s\right) - b, \pi_2\left(0, r^T; s, s\psi\right)$
N	$\pi_1\left(0, r^T; s, s\psi\right), \pi_2\left(r^T, 0; s\psi, s\right) - b$	$\frac{s}{9}, \frac{s}{9}$

[27] Note that in constrast, if negative royalties were not allowed, CS and SW would be higher under complete technology diffusion: $SW^{EL} - SW^T < 0$ and $CS^{EL} - CS^T < 0$.

4 Vertical integration (internal patent holder)

We have considered, so far, the case of an *external innovator*, that is, the U firm does not sell the final good in the D market. Suppose now that the U producer and one of the two D firms, say firm 2, merge, in this case the vertically integrated (VI) firm is an internal patent holder, that is a firm in the market producing with the new input. Note that as a consequence of being a unique entity it is not possible for the VI patent holder to replicate the deal with vertical separation because the upstream subsidiary cannot commit to shut down its downstream subsidiary. Therefore, its profit consists of two parts: the profit from selling the high quality final good 2 and the profit from selling the new input to the rival D firm 1 (if it decides to license).

We consider the following three-stage game: first, the patent holder (the VI firm) offers a contract to the rival firm 1, firm 1 decides whether to accept it and, finally, market competition takes place. Solving backwards, we find the Nash equilibrium for the internal patent holder subgame, that is, its optimal licensing incentives that we gather in the following Proposition.

Proposition 5 *The internal patent holder always sells the innovation to the rival firm. The optimal contract is the two-part tariff:*

$$r_{VI} = \frac{s\left(\psi\left(4\psi-1\right)-3\psi\sqrt{\psi}\right)}{2\left(4\psi-1\right)} > 0, F_{VI} = 0.$$

Proof. See Appendix (8.7). ■

Proposition 5 concerns the firm's incentive to license the product innovation thus preferring a homogeneous high quality good market rather than a vertically differentiated market in which the internal patentee would be the high quality firm competing with a low quality producer. The intuition relies on Bonanno (1986) where the author investigates the effect of the type of competition (Bertrand vs Cournot) on firms' incentives to (vertically) product differentiate. He shows that (with zero production costs) the two firms decide to produce a homogeneous high quality product under Cournot competition and differentiated products under Bertrand competition.

The result of Proposition 5 points out the incentive for the VI innovator to partially foreclose the rival firm via a positive royalty. More precisely, the equilibrium royalty, r_{VI}, is increasing in ψ so that the larger is the innovation the lower is the quantity produced by the rival non-affiliate. In other words, as the innovation size ψ increases the rival's outside option decreases and, in turn, the larger is the market share that the internal patentee is able to gain. The result is that, the larger is ψ, the more the market equlibrium approaches a monopoly.

Arya and Mittendorf (2006) study the incentives of an internal patent holder to license an innovation on the final good in the presence of a monopolist input producer. In their model the decision to license the innovation implies giving up the monopoly power on the final good, that is, letting a Cournot rival firm enter the market. They find that the licensing incentive is positive as doing so the internal patent holder creates a "weak" rival (as the rival has the additional marginal cost equal to the per-unit royalty) that induces the external supplier to make better pricing conditions. We also find that the VI (input) innovator has incentive to license the new good to the rival. The reason is that through partial input foreclosure the innovator attains to extract some surplus from the rival firm that however would be active in the market independently of the licensing strategy.

Notice that the optimal licensing policies in case of non-negative royalties for both the external and the internal patent holder are in line with previous contributions on process innovations (see among others, Sen and Tauman (2007) studying optimal licensing of process innovations in a Cournot oligopoly with homogeneous goods). We contribute to this optimal licensing literature by confirming and extending their results to the case of product innovations.

5 Private and social profitability of vertical integration

We can now consider the merger profitability comparing the vertical integration scenario with the vertical separation scenario (i.e., internal *versus* external patent holder). We gather our results in the following Proposition.

Proposition 6 *Vertical integration is always privately profitable. However, it is welfare improving if and only if the innovation is sufficiently large.*

Proof. See Appendix (8.8). ∎

The private profitability result is in line with the new market foreclosure theory according to which VI allows the U monopolist to protect its monopoly power. More precisely Rey and Tirole (2007) point out that vertical integration is a device to restore the U monopoly power that cannot be exerted under vertical separation when contracts are unobservable. In our model, contracts are observable, however, due to the strategic outside option, the external patent holder has not incentive to implement the monopoly outcome. Another way to look at this result is that VI is a device for the patent holder to make price discrimination (given that it can sell the innovation at marginal cost to its own affiliate and at a positive price to the rival firm); this option is not available under VS because the patent holder cannot credibly commit to make different prices.[28]

As for the social profitability, *as long as negative royalties are not allowed*, for large innovations ($\psi > \overline{\psi}$), vertical integration induces complete technology diffusion that would not be privately profitable for an external patent holder (that prefers exclusive licensing). In contrast, for small innovations, when the

[28] Considering a scenario where first the patent holder offers a license to firm 1 via the contract $(r_1 = 0, F_1)$ and second the patent holder offers another license to firm 2 via the contract (r_2, F_2), we find that the royalty offered to the second firm is non-positive. This scenario is dominated by the optimal contracts that we find.

new input is sold to all firms in the market under vertical separation, vertical integration is socially unprofitable. In other words, VI entails an anticompetitive effect because the VI firm is able to (partially) foreclose the rival firm via a positive per-unit royalty. Nevertheless, as long as the external patent holder prefers exclusive licensing, in the case of large innovations, VI by promoting the innovation diffusion, has the effect of improving the quality in the market. This last effect, when present, always compensates the anticompetitive effect. Under *negative royalties* the qualitative results are the same. In contrast with the case of non-negative royalties, the technology diffusion does not depend on the innovation size: the external patent holder always prefers exclusive licensing, so that the market is vertically differentiated; whereas, the internal patent holder always sells the innovation to the rival, so that in the market there is a homogeneous good of high quality. Therefore, under vertical separation, the double marginalization problem disappears because the innovating firm is subsidized and the non-innovating firm buys the input from competitive firms: competition in the market is rather strong. In contrast, under VI competition in the market is rather mild but there is a market quality improvement. The latter positive effect prevails whenever the innovation size is large enough.

We conclude that *allowing for negative royalties* does not affect the market outcome as, under both negative and non-negative royalties, the patent holder chooses to vertically integrate with either firm and to sell the innovation to the rival in the market. Figure 1 shows VI and licensing as the unique subgame perfect Nash equilibrium of the full game.

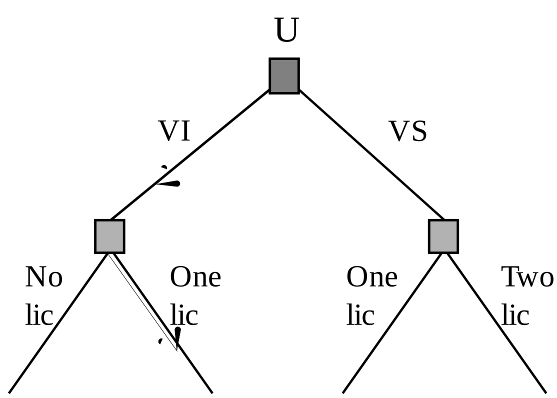

Figure 1: game tree

Under *non-negative royalties*, it is worth comparing these results with Sandonís and Faulí-Oller (2006) that consider non-drastic process innovations in a horizontally differentiated Cournot duopoly and study the patentee's incentives to merge with either firm in the market. They show that the merger is privately profitable for small innovations and it is welfare improving for large innovations. They argue that all profitable mergers are welfare detrimental. In our model the merger is profitable for any innovation size ψ and the profitability of VI increases with the innovation size because of the input foreclosure incentive pointed out in Proposition 5. This input foreclosure incentive does not take place in Sandonìs and Faulì-Oller (2006), where the VI firm gains from the market served by the rival firm, i.e., the VI firm prefers the duopoly (with very differentiated goods) rather than a quasi-monopoly. This explains why, in our model, VI dominates VS also for high values of ψ. In sum, for the private profitability of VI, the key difference of our results with respect to Sandonìs and Faulì-Oller (2006) is product differentiation; indeed in their framework, when goods are homogeneous VI becomes profitable for any innovation size.

6 Discussion

We next discuss the main assumptions of our model.

Production costs. We assume that, apart from the fixed cost for inventing the new input, production costs for the high quality input are equal to zero. This is a simplifying assumption that allows us to keep neat our results. It is also in line with previous contributions on licensing, see Li and Wang (2010, p. 520) and in line with previous contributions on vertical differentiation, see among others, Motta (1993, p. 114), Ishibashi and Matsushima (2009, p. 138). However one may wonder what happens if variable production costs are positive. To this aim we distinguish two cases. First, production costs that only depend on the quality, in the absence of quality choice, would not change the diffusion incentives as they should be sustained under both exclusive licensing and complete technology diffusion. Secondly, production costs that depend on the quantity would clearly change the quantitative results by affecting the equilibrium contracts. For instance, positive constant marginal costs for the high quality input would be passed through the per-unit royalty, without affecting our qualitative results, even though the negative royalties case would be replaced by a royalty lower than the marginal cost (in line with Milliou and Petrakis, 2007).

Three downstream firms. In the baseline model we consider a duopoly in the D market. This is a quite usual assumption as well as a benchmark in the literature, see, among others, Sandonís and Faulí-Oller (2006), Milliou and Petrakis (2007), Liao and Sen (2005). Hence, our approach, on one hand, allows us to get interesting and clear results on the licensing as well as on the vertical integration incentives; on the other hand, it allows us to draw the proper comparisons with the existing contributions.

Nevertheless, in order to understand whether the result that vertical integration facilitates innovation diffusion is robust, we extend the model considering the case of three firms in the D market. Formal details are provided in the Supplementary material. We next provide our results. Under vertical separation, that is, as long as the patent holder stays out of the market, whenever negative royalties are allowed, the patent holder optimally sells two licensees (partial technology diffusion). If negative royalties are not allowed, partial technology diffusion prevails again unless the innovation size is sufficiently low, in this case complete technology diffusion takes place. As for vertical integration, we distinguish two cases: either the U innovator merges with one out of three firms or the U innovator merges with two out of three firms. In the first scenario, complete technology diffusion prevails unless, under negative royalties, the innovation size is sufficiently small in which case partial technology diffusion takes place (the VI firm sells only one license); in the second scenario, which coincides with the VI scenario of our baseline model, the innovation is always sold to the rival non-affiliate. Therefore, *the innovation diffusion incentives are in line with the baseline model in that complete technology diffusion (resp. partial technology diffusion) is more likely to occur under vertical integration (resp. vertical separation).* However, in this extension, vertical integration lacks the profitability property. More precisely, vertical integration is not privately nor socially profitable anymore. The explanation is twofold. On one hand, the profitability of vertical separation increases because the D firms' outside options decrease due to the tougher competition. On the other hand, vertical integration with one out of three firms makes the (quasi-) monopoly outcome unprofitable because there are now two non-affiliate firms to compensate. Moreover, vertical separation is preferred to vertical integration with two out of three firms because this VI scenario entails a vertical as well as a horizontal merger, so that the positive effect coming from the internalization of the vertical externality does not compensate the negative effect of internalizing the horizontal externality.

As for the social welfare, VI with one out of three firms is never socially profitable. The external patent holder sells at least two licenses, so that in the market we have either two innovating firms competing with one non-innovating firm or three innovating firms. The main consequence when moving from VS to this scenario of VI is that, even though the innovation is at least as diffused as under VS, the VI firm is able to partially foreclose the non-affiliate firms. So that the anticompetitive effect of VI overcompensates the positive effect in terms of quality improvement. A fortiori, VI with two out of three firms is never socially profitable. Indeed, in this case VI implies one less firm in the market, without any positive effect in terms of quality improvement.

7 Conclusion

We have analysed the optimal licensing strategy of an upstream input innovator producing a new input which improves the quality of the final goods. We have considered a duopoly downstream market and two-part tariff contracts with non-negative fixed fees and either non-negative per-unit royalties or per-unit subsidies. We have shown that, under Cournot competition complete technology diffusion takes place and the innovator always prefers to be inside the market as the vertical merger with either downstream firm is always privately profitable. It is also welfare improving for large innovations. We thus show that vertical integration can be welfare improving with respect to vertical separation by inducing innovation diffusion.[29]

In the licensing game we have assumed that firms' outside option in case of an external patent holder is such that if the firm does not get the innovation the rival firm does. This means that the outside option depends on the royalty rate. Things are much different, in particular under complete technology diffusion, if we assume that either both firms get the innovation or neither does.[30] Indeed, in this case the outside option is constant with respect to the per-unit royalty so that the upstream innovator, via a two-part tariff contract, is able and finds it profitable to implement the monopoly outcome.[31] Clearly, the vertical integration would not be privately profitable anymore. In contrast, vertical integration would result to be welfare improving as a quasi monopoly would be better than a monopoly.

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[30] Somewhat in line with Gonzales and Ayala (2012) this could be a scenario where the downstream firms act cooperatively when purchasing the new input from the upstream innovator.

[31] This result has been proved by Inderst and Shaffer (2009) studying optimal two-part tariff contracts in vertical relations with independent asymmetric firms. Li and Wang (2010) obtains the same result for the specific framework of patent licensing.

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8 Appendix

8.1 Exclusive licensing subgame

Suppose that the innovator offers only one license, so that only one D firm adopts the new input. Firm 1 does not get the new input and produces a final good of quality $s_1 = s$ at price p_1; firm 2 adopts the new input and produces a final good of quality $s_2 = \psi s > s$, at price p_2 with $s_2 - s_1 = s(\psi - 1)$. Assuming that both firms stay active in the market, the demands for the goods are defined in (5) and (6). D firms' profits are:

$$\begin{aligned} \pi_1 &= p_1 q_1, \\ \pi_2 &= (p_2 - r_2) q_2 - F_2. \end{aligned}$$

In order to find the Cournot equilibrium, we assume that the demands for both goods are positive (formally, this means that $0 < \underline{\theta} = \frac{p_1}{s} < \widehat{\theta} = \frac{p_2 - p_1}{s(\psi-1)} < 1$), we derive the corresponding candidate equilibrium and we finally check whether this is the effective equilibrium.[32] *Cournot competition* leads to the following third stage quantity and price equilibrium:

$$\begin{aligned} q_1(0, r_2; s, s\psi) &= \frac{s\psi + r_2}{s(4\psi - 1)}, \\ q_2(r_2, 0; s\psi, s) &= \frac{(2s\psi - s - 2r_2)}{s(4\psi - 1)}, \\ p_1(0, r_2; s, s\psi) &= \frac{(s\psi + r_2)}{(4\psi - 1)}, \\ p_2(r_2, 0; s\psi, s) &= \frac{(2\psi - 1)(s\psi + r_2)}{(4\psi - 1)}, \end{aligned}$$

[32] See the inequalities at page 25 and the ones at page 26. This way of proceeding is standard in models of vertical differentiation (see, among others, Wauthy 1996).

with $q_2(r_2, 0; s\psi, s) > 0 \iff \frac{s(2\psi-1)}{2} > r_2$. Firm1 profit is then:

$$\pi_1(0, r_2; s, s\psi) = \frac{(s\psi + r_2)^2}{(4\psi - 1)^2 s} \tag{8}$$

As for firm 2:

$$\pi_2(r_2, 0; s\psi, s) = \frac{\psi(s - 2s\psi + 2r_2)^2}{(4\psi - 1)^2 s}.$$

The U firm chooses the two-part tariff contract for firm 2 (r_2, F_2) such that:

$$\begin{aligned} & \max_{r_2, F_2} \Pi_U \\ s.t. r_2 \quad \leq & \quad \frac{s(2\psi - 1)}{2} \\ 0 \quad \leq & \quad F_2 \leq \pi_2(r_2, 0; s\psi, s) - \pi_2(0, r_2; s, s\psi) \end{aligned}$$

with $\Pi_U = \frac{(2s\psi - s - 2r_2)}{s(4\psi - 1)} r_2 + F_2 - f$ and

$$\pi_2(0, r_2; s, s\psi) = \frac{(s\psi + r_2)^2}{(4\psi - 1)^2 s}, \tag{9}$$

if we reasonably assume that $r_1 = r_2$ (D firms are symmetric so that the equilibrium royalty set by the U innovator would be equal for either D firm). The first constraint comes from the non-negativity of q_2 and the second constraint (binding at equilibrium) ensures that firm 2 has the incentive to get the license rather than the outside option, that is not buying the innovation given that the rival firm 1 would get it.[33] The maximization problem thus becomes

$$\max_{r_2} \left[\frac{(2s\psi - s - 2r_2)}{s(4\psi - 1)} r_2 + \left(\frac{\psi(s - 2s\psi + 2r_2)^2}{(4\psi - 1)^2 s} - \frac{(s\psi + r_2)^2}{(4\psi - 1)^2 s} \right) \right]$$

This objective is concave in r_2 and the maximum is $r_2 = -\frac{s}{2}$. Under *non-negative royalties*, the solution is a contract such that (superscript EL stands for exclusive licensing):

$$r_2^{EL} = 0, F_2^{EL} = \frac{(\psi - 1) s\psi}{(4\psi - 1)}.$$

Equilibrium quantities and prices are:

$$\begin{aligned} q_1^{EL} &= (4\psi - 1)^{-1} \psi, q_2^{EL} = \frac{(2\psi - 1)}{(4\psi - 1)}, Q^{EL} = \frac{2\psi}{4\psi - 1} < 1 \\ p_1^{EL} &= \frac{\psi s}{(4\psi - 1)}, p_2^{EL} = \frac{(2\psi - 1)\psi s}{(4\psi - 1)}, \\ \widehat{\theta}_{EL} &= 2\frac{\psi}{(4\psi - 1)} \end{aligned}$$

[33] Note that in case of FR policy, the outside option of firm 2 is to buy the innovation knowing that if it does not get it, nobody will.

Equilibrium profits are:

$$\pi_1^{EL}(s, s\psi) = \pi_2^{EL}(s\psi, s) = \frac{\psi^2 s}{(4\psi-1)^2}$$
$$\Pi_U^{EL} = \frac{(\psi-1)\, s\psi}{(4\psi-1)} - f. \tag{10}$$

Producer surplus, consumer surplus and social welfare are:

$$\begin{aligned} PS^{EL} &= \tfrac{\left(4\psi^2-3\psi+1\right)s\psi}{(4\psi-1)^2} - f, \\ CS^{EL} &= \tfrac{\left(\psi+4\psi^2-1\right)s\psi}{2(4\psi-1)^2}, \\ SW^{EL} &= \tfrac{\left(12\psi^2-5\psi+1\right)s\psi}{2(4\psi-1)^2} - f. \end{aligned}$$

Π_U^{EL} defined in (10) is the U patent holder equilibrium profit under exclusive licensing, when selling via a two-part tariff, which reduces to a fixed fee, the new input to only one D firm. We check whether at equilibrium, the conditions for both firms to stay in the market are satisfied, and we find that they are:

$$\begin{aligned} p_2 &< p_1 + s(\psi-1) \iff \frac{(\psi-1)(2\psi-1)\, s}{(1-4\psi)} < 0, \forall\psi, \\ p_2 &> p_1\psi \iff \frac{(\psi-1)\, s\psi}{(4\psi-1)} > 0, \forall\psi. \end{aligned}$$

Consider now the case of *negative royalties*, the optimal contract under exclusive licensing is then such that:

$$r_2^{Neg} = -\frac{s}{2}, F_2^{Neg} = \frac{\left(4\psi^2+1\right)s}{4(4\psi-1)}. \tag{11}$$

Equilibrium quantities, prices, firms' profits, the external patentee's equilibrium profit, producer surplus, consumer surplus and social welfare are (superscript $ELneg$ stands for exclusive licensing and negative royalties):

$$\begin{aligned} q_1^{ELneg}(0, r_2; s, s\psi) &= \frac{(2\psi-1)}{2(4\psi-1)}, q_2^{ELneg} = 2\frac{\psi}{(4\psi-1)}, Q^{ELneg} = \frac{6\psi-1}{2(4\psi-1)} < 1 \\ p_1^{ELneg} &= \frac{(2\psi-1)\, s}{2(4\psi-1)} < p_2^{ELneg} = \frac{s(2\psi-1)^2}{2(4\psi-1)} \\ \pi_2^{ELneg}(s\psi, s) &= \frac{(2\psi-1)^2 s}{4(4\psi-1)^2} = \pi_1^{ELneg}(s, s\psi) \end{aligned}$$

$$
\begin{aligned}
\Pi_U^{ELneg} &= \frac{(2\psi-1)^2 s}{4(4\psi-1)} - f, && (12)\\
PS^{ELneg} &= \tfrac{(2\psi-1)^2 s}{4(4\psi-1)} + 2\tfrac{(2\psi-1)^2 s}{4(4\psi-1)^2} - f = \tfrac{(2\psi-1)^2(4\psi+1)s}{4(4\psi-1)^2} - f,\\
CS^{ELneg} &= \tfrac{\left(20\psi^2-12\psi+16\psi^3+1\right)s}{8(4\psi-1)^2},\\
SW^{ELneg} &= \tfrac{\left(48\psi^3-4\psi^2-12\psi+3\right)s}{8(4\psi-1)^2} - f.
\end{aligned}
$$

$$
\widehat{\theta}_{ELneg} = \frac{(2\psi-1)}{(4\psi-1)}
$$

Note that under both cases of non-negative and negative royalties, the D firms are worse off with respect to the status quo.[34] However, if either firm thinks that the rival is not making an offer, then this firm will have the incentive to make a slightly positive offer and get the innovation and this reasoning holds up to the outside option. Note also that, as one could expect $\Pi_U^{EL} - \Pi_U^{ELneg} = -\frac{s}{4(4\psi-1)} < 0$, the patent holder prefers not being constrained to non-negative royalties. Also, from the industry point of view, $\Pi^{EL} - \Pi^{ELneg} = \frac{s}{4(4\psi-1)} > 0$, given that under negative royalties production is subsidized. However, computing the joint profits of the patent holder and the licensee under vertical separation we find that $\Pi_U^{EL} + \pi_2^{EL}(s\psi, s) = \Pi_U^{ELneg} + \pi_2^{ELneg}(s\psi, s)$, that is the two separated subjects, jointly, are indifferent between negative and non-negative royalties. As for consumers and social welfare we find that:

$$
\begin{aligned}
CS^{EL} - CS^{ELneg} &= \tfrac{\left(\psi+4\psi^2-1\right)s\psi}{2(4\psi-1)^2} - \tfrac{\left(20\psi^2-12\psi+16\psi^3+1\right)s}{8(4\psi-1)^2} = -\frac{1}{8}s < 0,\\
SW^{EL} - SW^{ELneg} &= \tfrac{\left(12\psi^2-5\psi+1\right)s\psi}{2(4\psi-1)^2} - \tfrac{\left(48\psi^3-4\psi^2-12\psi+3\right)s}{8(4\psi-1)^2} = -\frac{(4\psi-3)s}{8(4\psi-1)} < 0.
\end{aligned}
$$

Finally, we verify that, also at this equilibrium, the conditions for both firms to stay in the market are satisfied:

$$
\begin{aligned}
p_2 &< p_1 + s(\psi-1) \iff -2s\psi\frac{\psi-1}{4\psi-1} < 0, \forall\psi,\\
p_2 &> p_1\psi \iff \frac{1}{2}s(2\psi-1)\frac{\psi-1}{4\psi-1} > 0, \forall\psi.
\end{aligned}
$$

8.2 Complete technology diffusion subgame

Suppose the U firm decides to offer two licenses, $m = 2$. In this case, the patent holder will also set a minimum bid b, otherwise no firm would make a positive offer because each firm is guaranteed to have a license irrespective of its bid.

[34] Formally: $\frac{\psi^2 s}{(4\psi-1)^2} - \frac{s}{9} = -\frac{1}{9}s(7\psi-1)\frac{\psi-1}{(4\psi-1)^2} < 0$ and $\frac{(2\psi-1)^2 s}{4(4\psi-1)^2} - \frac{s}{9} = -\frac{1}{36}s(2\psi+1)\frac{14\psi-5}{(4\psi-1)^2} < 0$.

When the policy $m=2$, r and b is offered, in equilibrium each firm will offer this minimum bid, b.[35] The U firm maximization problem is:

$$\max_{F_1,r_1,F_2,r_2}\left\{r_1 q_1\left(r_1,r_2;s\psi,s\psi\right)+r_2 q_2\left(r_1,r_2;s\psi,s\psi\right)+F_1+F_2-f\right\} \tag{13}$$
$$s.t.\pi_1\left(r_1,r_2;s\psi,s\psi\right)-F_1\geq\pi_1\left(0,r_2;s,s\psi\right)$$
$$\pi_2\left(r_2,r_1;s\psi,s\psi\right)-F_2\geq\pi_2\left(0,r_1;s,s\psi\right)$$
$$F_1\geq 0, F_2\geq 0$$

where $\pi_1\left(0,r_2;s,s\psi\right)$ is defined in (8) for firm 2. Here the outside option for each firm is not buying the new input given that the rival firm does. The optimal fixed fee corresponding to b is $\pi_i\left(r_i,r_j;s\psi,s\psi\right)-\pi_i\left(0,r_j;s,s\psi\right)$. $\pi_i\left(r_i,r_j;s\psi,s\psi\right)$ and $q_i\left(r_i,r_j;s\psi,s\psi\right)$ denote the third stage equilibrium D firm i profit and quantity when both firms produce the high quality good, namely:

$$\pi_i\left(r_i,r_j;s\psi,s\psi\right)=\frac{\left(s\psi-2r_i+r_j\right)^2}{9\psi s}, \tag{14}$$

$$q_i\left(r_i,r_j;s\psi,s\psi\right)=\frac{1}{3s\psi}\left(s\psi-2r_i+r_j\right). \tag{15}$$

$$p_i\left(r_i,r_j;s\psi,s\psi\right)=s\psi\left(1-\frac{1}{3s\psi}\left(s\psi-2r_i+r_j\right)-\frac{1}{3s\psi}\left(s\psi-2r_j+r_i\right)\right)$$

As the two constraints are binding at equilibrium, we have

$$\begin{aligned}F_1\left(r_1,r_2\right) &= \pi_1\left(r_1,r_2;s\psi,s\psi\right)-\pi_1\left(0,r_2;s,s\psi\right),\\ F_2\left(r_1,r_2\right) &= \pi_2\left(r_2,r_1;s\psi,s\psi\right)-\pi_2\left(0,r_1;s,s\psi\right),\end{aligned}$$

with $\pi_i\left(0,r_j;s,s\psi\right)=\frac{(s\psi+r_j)^2}{(4\psi-1)^2 s}$. The maximization problem, thus becomes:

$$\max_{r_1,r_2}\left\{r_1 q_1\left(r_1,r_2;s\psi,s\psi\right)+r_2 q_2\left(r_2,r_1;s\psi,s\psi\right)+F_1\left(r_1,r_2\right)+F_2\left(r_1,r_2\right)-f\right\}.$$

Solving this problem we find that under non-negative royalties, the optimal contract is:

$$\begin{cases} r_1=r_2=r^T=\frac{\left(s\psi-26s\psi^2+16s\psi^3\right)}{64\psi^2-14\psi+4},\ F^T\left(r^T,r^T\right)=\frac{\left(248\psi^2-17\psi-272\psi^3+256\psi^4+1\right)s\psi}{4\left(32\psi^2-7\psi+2\right)^2} \text{ if } \psi\geq\psi^T, \\ r_1=r_2=0,\ F^T\left(0,0\right)=\frac{(\psi-1)(16\psi-1)s\psi}{9(4\psi-1)^2} \text{ if } \psi<\psi^T. \end{cases}$$

where $r^T\geq 0 \iff \psi\geq\psi^T=1.5856$. This means that when the innovation is small the inventor's incentive is to set a per-unit price as low as possible, that is the optimal contract is a fixed fee. In other words, given the positive outside option of producing the low quality good at zero marginal cost, for small innovations the inventor cannot set a positive per unit royalty. In contrast for large innovations we have a positive per-unit royalty.

[35] In case the innovator offers two licenses, the AR and the FR policy coincide. See Liao and Sen (2005) for details.

If we allow for *negative royalties*, the optimal contract is the first line of (7) for any innovation size. Note that for a small innovation size ($\psi < \psi^T$), the U monopolist is willing to subsidize the D production, however in this case the explanation is not the incentive to make the D affiliates more aggressive but the presence of the outside option: each D firm can always produce the low quality good at zero marginal cost and make positive profit. To induce them to buy the small innovation the inventor cannot set a positive per unit royalty.

Equilibrium magnitudes are for $\psi \geq \psi^T$, and for any innovation size under negative royalties:

$$q^T(s\psi) = \frac{\left(4\psi+16\psi^2+1\right)}{2(32\psi^2-7\psi+2)}, Q^T = \frac{\left(16\psi^2+4\psi+1\right)}{(32\psi^2-7\psi+2)}$$
$$p^T = \frac{\left(16\psi^2-11\psi+1\right)\psi s}{(32\psi^2-7\psi+2)}$$
$$\pi^T(s\psi) = \frac{\left(4\psi+16\psi^2+1\right)^2 s\psi}{4(32\psi^2-7\psi+2)^2}$$

with $Q^T < 1$ as $\left(16\psi^2+4\psi+1\right) - \left(32\psi^2-7\psi+2\right) = 11\psi - 16\psi^2 - 1 < 0$, which also implies $p^T < s\psi$;

$$\Pi_U^T(s\psi) = \frac{\left(16\psi^2-16\psi+1\right)s\psi}{2(32\psi^2-7\psi+2)} - f, \tag{16}$$
$$PS^T = \frac{\left(16\psi^2-11\psi+1\right)\left(4\psi+16\psi^2+1\right)s\psi}{(32\psi^2-7\psi+2)^2} - f,$$
$$CS^T = \frac{\left(4\psi+16\psi^2+1\right)^2 s\psi}{2(32\psi^2-7\psi+2)^2},$$
$$SW^T = \frac{3}{2}\frac{\left(16\psi^2-6\psi+1\right)\left(4\psi+16\psi^2+1\right)s\psi}{(32\psi^2-7\psi+2)^2} - f.$$

Whereas for $\psi < \psi^T$, in case of non-negative royalties, equilibrium magnitudes are:

$$q_d^T(s\psi) = \frac{1}{3}, Q_d^T = \frac{2}{3},$$
$$p_d^T(s\psi) = \frac{s\psi}{3},$$
$$\pi_d^T(s\psi) = \frac{s\psi}{9}.$$

Notice that at equilibrium it always holds that $p_d^T(s\psi) = \frac{s\psi}{3} < s\psi$.

$$\Pi_{Ud}^T(s\psi) = 2F^T(0,0) - f = 2\frac{(\psi-1)(16\psi-1)s\psi}{9(4\psi-1)^2} - f, \tag{17}$$
$$PS_d^T = \frac{2}{9}\psi s - f, CS_d^T = \frac{2}{9}\psi s,$$
$$SW_d^T = \frac{4}{9}\psi s - f.$$

8.3 Proof of Proposition 1

Given the equilibrium analysis of Sections (8.1) and (8.2), direct comparisons of (12), (10), (16) and (17) show that:

$$\Pi_U^{EL} - \Pi_U^T = \begin{cases} \text{for } \psi \geq \psi^T, \ \frac{(\psi-1)s\psi}{(4\psi-1)} - \frac{\left(16\psi^2-16\psi+1\right)s\psi}{2(32\psi^2-7\psi+2)} = s\psi\frac{\left(2\psi^2-2\psi-3\right)}{2(32\psi^2-7\psi+2)(4\psi-1)} > 0 \iff \psi > 1.8 \equiv \overline{\psi} \\ \text{for } \psi < \psi^T, \ \frac{(\psi-1)s\psi}{(4\psi-1)} - 2\frac{(\psi-1)(16\psi-1)s\psi}{9(4\psi-1)^2} = \frac{(4\psi-7)(\psi-1)s\psi}{9(4\psi-1)^2} < 0 \end{cases}$$

$$\Pi_U^{ELneg} - \Pi_U^T = \frac{(2\psi-1)^2 s}{4(4\psi-1)} - \frac{\left(16\psi^2-16\psi+1\right)s\psi}{2(32\psi^2-7\psi+2)} = \frac{\left(28\psi^2-13\psi+4\psi^3+2\right)s}{4(32\psi^2-7\psi+2)(4\psi-1)} > 0.$$

8.4 Proof of Corollary 3

Straightforward comparisons with respect to the status quo show that the consumer surplus is higher: $CS^T - CS > 0$, $CS_d^T - CS > 0$, $CS^{EL} - CS > 0$, $CS^{ELneg} - CS > 0$; as for social welfare, as long as f is not too large $SW^T - SW > 0$, $SW_d^T - SW > 0$, $SW^{EL} - SW > 0$, $SW^{ELneg} - SW > 0$; as for the licensees, they are worse off, $\pi_i^{EL}(s\psi, s) - \pi_i < 0$, $\pi_i^{ELneg}(s\psi, s) - \pi_i < 0$, $\pi^T(s\psi) - F^T\left(r^T, r^T\right) - \pi_i < 0$, and $\pi_d^T(s\psi) - F^T(0,0) - \pi_i < 0$.

8.5 Proof of Corollary 4

As already pointed out, $SW^{EL} < SW^{ELneg}$, this comparison is relevant for $\psi > \overline{\psi}$. Also:

$$\text{for } \psi \geq \psi^T, \begin{cases} SW^{ELneg} - SW^T = \frac{\left(971\psi^2-144\psi-3644\psi^3+7932\psi^4-9680\psi^5+5120\psi^6+12\right)}{8(32\psi^2-7\psi+2)^2(4\psi-1)^2} s > 0 \\ CS^{ELneg} - CS^T = s\frac{-80\psi+593\psi^2-3068\psi^3+10\,004\psi^4-18\,416\psi^5+13\,312\psi^6+4}{8(4\psi-1)^2(-7\psi+32\psi^2+2)^2} > 0 \\ PS^{ELneg} - PS^T = -s\frac{32\psi-189\psi^2+288\psi^3+1036\psi^4-4368\psi^5+4096\psi^6-4}{4(4\psi-1)^2(-7\psi+32\psi^2+2)^2} < 0 \end{cases},$$

$$\text{for } \psi < \psi^T, \begin{cases} SW^{ELneg} - SW_d^T = -\frac{\left(140\psi-220\psi^2+80\psi^3-27\right)}{72(4\psi-1)^2} s > 0 \\ CS^{ELneg} - CS_d^T = -s\frac{124\psi-308\psi^2+112\psi^3-9}{72(4\psi-1)^2} > 0 \\ PS^{ELneg} - PS_d^T = s(2\psi+1)\frac{-26\psi+8\psi^2+9}{36(4\psi-1)^2} < 0 \end{cases}.$$

8.6 Vertical integration subgame

Consider the quantity competition between the VI firm and firm 1 producing the high quality final good. The VI firm has zero variable production costs as the new input is transferred at the marginal cost $c_2 = 0$, whereas firm 1 incurs marginal cost r_1. The third stage equilibrium quantities and profits are:

$$q_{VI}\left(0, r_1; s\psi, s\psi\right) = \frac{1}{3s\psi}\left(s\psi + r_1\right)$$
$$q_1\left(r_1, 0; s\psi, s\psi\right) = \frac{1}{3s\psi}\left(s\psi - 2r_1\right) \geq 0 \iff r_1 \leq \frac{s\psi}{2}$$
$$p_{VI} = s\psi\left(1 - q_{VI}\left(0, r_1; s\psi, s\psi\right) - q_1\left(r_1, 0; s\psi, s\psi\right)\right) = \frac{1}{3}\left(s\psi + r_1\right)$$
$$\pi_{VI}\left(0, r_1; s\psi, s\psi\right) = \frac{\left(s\psi + r_1\right)^2}{9s\psi} - f$$
$$\pi_1\left(r_1, 0; s\psi, s\psi\right) = \frac{\left(s\psi - 2r_1\right)^2}{9\psi s}$$

where $q_{VI}\left(0, r_1; s\psi, s\psi\right)$ and $q_1\left(r_1, 0; s\psi, s\psi\right)$ are obtained from expression (15) substituting properly r_i and r_j; $\pi_{VI}\left(0, r_1; s\psi, s\psi\right)$ and $\pi_1\left(r_1, 0; s\psi, s\psi\right)$ are obtained from expression (14) substituting properly r_i and r_j. The VI firm offers firm 1 the two-part tariff contract (r_1, F_1) such that:[36]

$$\max_{r_1, F_1}\left\{\pi_{VI}\left(0, r_1; s\psi, s\psi\right) + r_1 q_1\left(r_1, 0; s\psi, s\psi\right) + F_1\right\}$$
$$s.t.\pi_1\left(r_1, 0; s\psi, s\psi\right) - F_1 \geq \pi_1\left(0, 0; s, s\psi\right)$$
$$r_1 \leq \frac{s\psi}{2},$$

where $\pi_1\left(0, 0; s, s\psi\right) = \frac{\psi^2 s}{(4\psi-1)^2}$, obtained from (8) is firm 1 outside option. As the first constraint is binding at equilibrium, we have:

$$\max_{r_1}\left\{\pi_{VI}\left(0, r_1; s\psi, s\psi\right) + r_1 q_1\left(r_1, 0; s\psi, s\psi\right) + \pi_1\left(r_1, 0; s\psi, s\psi\right) - \pi_1\left(0, 0; s, s\psi\right)\right\}$$

The optimal contract is then:

$$r_1^* = s\frac{\psi}{2}, F_1^* = -\frac{\psi^2 s}{\left(4\psi - 1\right)^2}.$$

If we let the VI firm to set negative fees, the vertical merger implements the monopoly outcome by inducing the nonintegrated firm to produce a nil quantity (foreclosure) and compensating it for the outside option. Equilibrium magni-

[36] In the following maximization problem we do not restrict the VI firm to set nonnegative fees. This allows us to make clear its incentives. We next solve the maximization problem constrained to nonnegative fees.

tudes are:

$$q_{VI}\left(0, r_1^*; s\psi, s\psi\right) = \frac{1}{2} = q^m, q_1\left(r_1^*, 0; s\psi, s\psi\right) = 0,$$

$$p_{VI} = \frac{1}{2}\psi s = p^m,$$

$$\pi_{VI}\left(0, r_1^*; s\psi, s\psi\right) = \frac{1}{4}\psi s, \pi_1\left(r_1^*, 0; s\psi, s\psi\right) - F_1^* = \frac{\psi^2 s}{\left(4\psi - 1\right)^2},$$

$$\Pi_{VI} = \pi_{VI}\left(0, r_1^*; s\psi, s\psi\right) - f + r_1^* q_1\left(r_1^*, 0; s\psi, s\psi\right) + F_1^* = \frac{1}{4}\psi s - \frac{\psi^2 s}{\left(4\psi - 1\right)^2} - f.$$

However negative fees would be clearly held to be illegal by antitrust authorities. It is clear from the analysis above that the VI firm wants to restrict as much as possible the quantity produced by the non affiliate firm so as to (at least) partially internalize the vertical externality.

If the VI firm is constrained to nonnegative fees, it will optimally let the nonaffiliate firm to produce a positive quantity as low as possible (up to its outside option) $q_1\left(r_1\right) : \pi_1\left(r_1\right) = \frac{\psi^2 s}{(4\psi-1)^2}$. Given the Cournot equilibrium quantities, we have

$$\left(\frac{1}{3}\left(s\psi + r_1\right) - r_1\right)\frac{1}{3s\psi}\left(s\psi - 2r_1\right) = \frac{\psi^2 s}{\left(4\psi - 1\right)^2}$$

$$\iff r_1 = \frac{\left(\psi s\left(4\psi - 1\right) - 3s\psi\sqrt{\psi}\right)}{2\left(4\psi - 1\right)}.$$

The optimal contract is then:

$$r_{VI} = s\frac{\left(\psi\left(4\psi - 1\right) - 3\psi\sqrt{\psi}\right)}{2\left(4\psi - 1\right)} > 0, F_{VI} = 0; \tag{18}$$

with $r_{VI} < s\frac{\psi}{2}$ for any ψ and $\frac{\partial}{\partial\psi}r_{VI} > 0$.[37] Equilibrium quantities and price are:

$$q_{VI}\left(0, r_{VI}; s\psi, s\psi\right) = \frac{\left(4\psi^2 - \psi\sqrt{\psi} - \psi\right)}{2\left(4\psi - 1\right)\psi},$$

$$q_1\left(r_{VI}, 0; s\psi, s\psi\right) = \frac{\sqrt{\psi}}{\left(4\psi - 1\right)},$$

$$Q_{VI} = q_{VI}\left(0, r_{VI}; s\psi, s\psi\right) + q_1\left(r_{VI}, 0; s\psi, s\psi\right) = \frac{\left(4\psi^2 - \psi + \psi^{\frac{3}{2}}\right)}{2\left(4\psi - 1\right)\psi},$$

$$p_{VI} = \frac{\psi\left(4\psi - \sqrt{\psi} - 1\right)s}{2\left(4\psi - 1\right)}.$$

[37] Note that $\lim_{\psi\to\infty} r_{VI} = \infty$ and $\lim_{\psi\to\infty} q_{VI} = 0$.

with $Q_{VI} < 1$ as $\left(4\psi^2 - \psi + \psi^{\frac{3}{2}}\right) - 2\left(4\psi - 1\right)\psi = -\psi\left(4\psi - \sqrt{\psi} - 1\right) < 0$. Equilibrium profits, CS, PS and SW are:

$$\pi_{VI}\left(0, r_{VI}; s\psi, s\psi\right) = \frac{\left(4\psi^2 - \psi\sqrt{\psi} - \psi\right)^2 s}{4\left(4\psi - 1\right)^2 \psi},$$

$$\pi_1\left(r_{VI}, 0; s\psi, s\psi\right) - F_{VI} = \frac{s\psi^2}{\left(4\psi - 1\right)^2},$$

$$\Pi_{VI} = \pi_{VI}\left(0, r_{VI}; s\psi, s\psi\right) - f + r_{VI}q_1\left(r_{VI}, 0; s\psi, s\psi\right) + F_{VI} = \frac{\left(16\psi^2 - 13\psi + 1\right)s\psi}{4(4\psi - 1)^2}, \tag{19}$$

$$CS_{VI} = \frac{\left(4\psi + \sqrt{\psi} - 1\right)^2 s\psi}{8(4\psi - 1)^2},$$

$$PS_{VI} = \frac{s\psi^2}{(4\psi - 1)^2} + \frac{\left(16\psi^2 - 13\psi + 1\right)s\psi}{4(4\psi - 1)^2},$$

$$SW_{VI} = \frac{\left(4\psi + \sqrt{\psi} - 1\right)^2 s\psi}{8(4\psi - 1)^2} + \frac{s\psi^2}{(4\psi - 1)^2} + \frac{\left(16\psi^2 - 13\psi + 1\right)s\psi}{4(4\psi - 1)^2}.$$

Note that the optimal contract is the same if we allow for negative royalties, as the VI firm has always incentive to set a positive royalty.

8.7 Proof of Proposition 5

The proof comes from the VI firm maximization problem, given that it could always decide not to sell the license and get profit equal to $\pi_{VI}\left(0, 0; s\psi, s\right) = \frac{(2\psi - 1)^2 s\psi}{(4\psi - 1)^2} < \Pi_{VI}$ defined in expression (19).

8.8 Proof of Proposition 6

As far as the private profitability is concerned, we wonder whether the patent holder prefers to stay out of the market or to vertically integrate with either firm given the outcome of the possible subgames previously analysed. We find that vertical integration is always privately profitable. Namely, comparing the profit under VI (Π_{VI}) with the joint profit under VS (the profit of the external patent holder plus the profit of either potential licensee), we obtain under *non-negative royalties*:

for $\psi > \overline{\psi}$, $\Pi_{VI} - \left(\Pi_U^{EL} + \pi_i^{EL}\left(s, s\psi\right)\right) = \frac{3(\psi - 1)s\psi}{4(4\psi - 1)^2} > 0$,

for $\psi \in \left[\psi^T, \overline{\psi}\right]$, $\Pi_{VI} - \left(\Pi_U^T + \pi^T\left(s\psi\right) - F^T\left(r^T, r^T\right)\right) = \frac{5}{4}s\psi^2 \frac{\left(256\psi^4 - 57\psi^2 - 112\psi^3 - 7\psi + 1\right)}{(32\psi^2 - 7\psi + 2)^2(4\psi - 1)^2} > 0$,

for $\psi < \psi^T$, $\Pi_{VI} - \left(\Pi_{Ud}^T + \pi^T\left(s\psi\right) - F^T\left(r^T, r^T\right)\right) = \frac{(\psi - 1)(16\psi - 1)s\psi}{36(4\psi - 1)^2} > 0$,

and under *negative royalties*:

$$\Pi_{VI} - \left(\Pi_U^{ELneg} + \pi_i\left(r_i, 0; s\psi, s\right)\right) = \frac{3(\psi - 1)s\psi}{4(4\psi - 1)^2} > 0.$$

As for the social profitability, we find that vertical integration is socially profitable for large innovations, whereas it is welfare detrimental for small innovations. Namely, we make the following comparisons, under non-negative royalties:

$$\text{for } \psi > \overline{\psi}, \begin{cases} SW_{VI} - SW^{EL} = \frac{\left(8\psi^{\frac{5}{2}} - 5\psi^2 - 2\psi^{\frac{3}{2}} - \psi\right)s}{8(4\psi-1)^2} > 0 \\ CS_{VI} - CS^{EL} = \frac{s\psi}{(4\psi-1)^2}\left(\frac{\left(4\psi+\sqrt{\psi}-1\right)^2}{8} - \frac{\left(\psi+4\psi^2-1\right)}{2}\right) > 0 \\ PS_{VI} - PS^{EL} = \frac{3(\psi-1)s\psi}{4(4\psi-1)^2} > 0 \end{cases}$$

$$\text{for } \psi \in \left[\psi^T, \overline{\psi}\right], \begin{cases} SW_{VI} - SW^{T} = \frac{s\psi\left(\sqrt{\psi}\left(45\psi - 180\psi^2 + 256\psi^3 - 2\sqrt{\psi} + 7\psi^{\frac{3}{2}} - 32\psi^{\frac{5}{2}} - 4\right)\left(32\psi^2 - 7\psi + 15\sqrt{\psi} - 60\psi^{\frac{3}{2}} + 2\right)\right)}{8\left(2\sqrt{\psi}+1\right)^2\left(2\sqrt{\psi}-1\right)^2(32\psi^2-7\psi+2)^2} \\ CS_{VI} - CS^{T} = s\psi\left(\frac{\left(4\psi+\sqrt{\psi}-1\right)^2}{8(4\psi-1)^2} - \frac{\left(4\psi+16\psi^2+1\right)^2}{2(32\psi^2-7\psi+2)^2}\right) < 0 \\ PS_{VI} - PS^{T} = -\frac{\left(1977\psi^2 - 253\psi - 4048\psi^3 + 1024\psi^4 + 4\right)s\psi^2}{4(32\psi^2-7\psi+2)^2(4\psi-1)^2} > 0 \end{cases}$$

$$\text{for } \psi < \psi^T, \begin{cases} SW_{VI} - SW_d^{T} = \psi s\left(\frac{\left(4\psi+\sqrt{\psi}-1\right)^2}{8(4\psi-1)^2} + \frac{\psi}{(4\psi-1)^2} + \frac{\left(16\psi^2-13\psi+1\right)}{4(4\psi-1)^2} - \frac{4}{9}\right) < 0 \\ CS_{VI} - CS_d^{T} = \left(\frac{\left(4\psi+\sqrt{\psi}-1\right)^2}{8(4\psi-1)^2} - \frac{2}{9}\right)\psi s < 0 \\ PS_{VI}^{C} - PS_d^{T} = \frac{(\psi-1)(16\psi-1)s\psi}{36(4\psi-1)^2} > 0 \end{cases}$$

And, under negative royalties:

$$\begin{aligned} SW_{VI} - SW^{ELneg} &= s\frac{\left(15\psi - 21\psi^2 - 2\psi^{\frac{3}{2}} + 8\psi^{\frac{5}{2}} - 3\right)}{8(4\psi-1)^2} > 0 \iff \psi > 5.85 \equiv \psi^* > \overline{\psi}, \\ CS_{VI} - CS^{ELneg} &= s\frac{\left(\left(4\psi+\sqrt{\psi}-1\right)^2\psi - \left(20\psi^2 - 12\psi + 16\psi^3 + 1\right)\right)}{8(4\psi-1)^2} < 0, \\ PS_{VI} - PS^{ELneg} &= s\frac{\left(\psi+3\psi^2-1\right)}{4(4\psi-1)^2} > 0. \end{aligned}$$